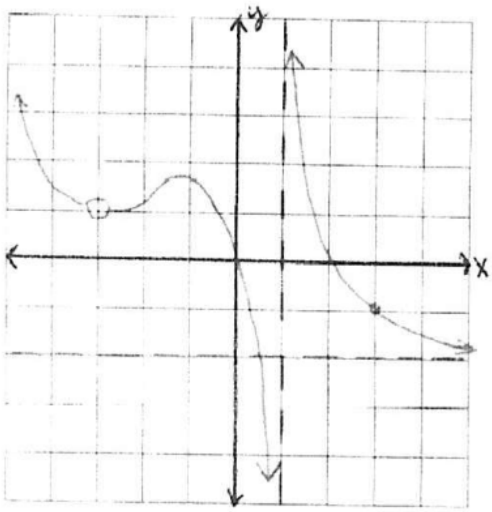
Name________________________________ Calculus
Review

This review sheet should NOT serve as your only review. You should review all notes and tests.

Questions 1 through 7 refer to the graph of $y = f(x)$ shown to the right.



1. $\lim_{x \to 1^-} f(x) =$

2. $\lim_{x \to 1^+} f(x) =$

3. $\lim_{x \to 1} f(x) =$

4. $\lim_{x \to -3} f(x) =$

5. $\lim_{x \to 3} f(x) =$

6. $\lim_{x \to -\infty} f(x) =$

7. $\lim_{x \to \infty} f(x) =$

For each of the following functions, use the definition of derivative to find $f'(x)$.

Recall: $f'(x) = \lim_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$

8 a) $f(x) = 2x^2 - 8x + 5$

b) $f(x) = \sqrt{x+2}$

Find the derivative of each of the following:

9. $f(x) = 8x + 2\sqrt[3]{x} - \frac{3}{x^3}$ 10. $f(x) = \sin(5x^3 + 2x)$ 11. $f(x) = \sqrt[3]{(5x^2 + 2x)^2}$

$f(x) = 8x + 2x^{1/3} - 3x^{-3}$

$f'(x) = 8 + \frac{2}{3}x^{-2/3} + 9x^{-4}$

12. Find the slope of the line tangent to $y = x(\cos(x))$ when $x = 0$.

$\boxed{y-6=-8(x+1)}$

13. Write the equation of the line tangent to $y = 3x^2 - 2x + 1$ when $x = -1$.

Point: $y = 3(-1)^2 - 2(-1) + 1$ $y' = 6x - 2$

$(-1, 6)$ $= 3 + 2 + 1$ $y' = 6(-1) - 2$

$= 6$ slope $= -8$

14. The following table records the values of $f, f', g, and\ g'$ at $x = 1$, $x = 2$, and $x = 3$.

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	2	1	2	3
2	5	4	3	4
3	0	6	-1	-2

If $n(x) = \frac{f(x)}{g(x)}$, $h(x) = f(g(x))$, find the value of each of the following: a) $n'(2)$ b) $h'(1)$

$n'(2) = \frac{g(2) \cdot f'(2) - f(2) \cdot g'(2)}{(g(2))^2} = \frac{3(4) - 5(4)}{3^2} = \frac{-8}{9}$

15. If $f(x) = \sqrt[3]{(x^2 - 2x - 1)^2}$, then $f'(0) = ?$

12) $y = \boxed{x}\,\boxed{\cos(x)}$ $x = 0$

Slope = Derivative

$y' = x(-\sin(x)) + 1\cos(x)$

$y' = -x\sin(x) + \cos(x)$

$= \cos(x) - x\sin(x)$

$= \cos(0) - \cancel{0(\sin(0))}$

$= 1$

Slope = 1

14) b) $h(x) = f(g(x))$ ← chain rule

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$h'(x) = f'(g(x)) \cdot g'(x)$

$h'(1) = f'(\underbrace{g(1)}) \cdot g'(1)$

$= f'(2) \cdot 3$

$= (4)(3) = 12$

16. Is $h(x)$ continuous for all real numbers? If so show why.

$$h(x)=\begin{cases} x+3, & x\le -2 \\ -x^2, & x>-2 \end{cases}$$

are they equal at $x=-2$?

$-2+3 = 1$

$-(-2)^2 = -4$

Not the same $\therefore$ Not continuous.

17. Evaluate $\lim_{x\to 2}\frac{x^2+3x-10}{x-2}$.

18. Evaluate $\lim_{x\to\infty}\frac{2x^3-3}{3x^3+25}$. $= \frac{2}{3}$

19. Find the derivative of the following:

a) $f(x)=e^{2x}\sin(3x)$

Differentiate!

b) $y=\frac{\ln(2x)}{\sqrt{x^2+5x}}$

$$= \frac{\ln(2x)}{(x^2+5x)^{1/2}} = \underbrace{\boxed{(\ln(2x))}}^{\text{1st}}\,\overbrace{\boxed{(x^2+5x)^{-1/2}}}^{\text{2nd}}$$

$$y' = \underbrace{(\ln(2x))}_{\text{First}}\underbrace{\left(-\tfrac{1}{2}\right)(x^2+5x)^{-3/2}\cdot(2x+5)}_{\text{derivative of 2nd}} + \underbrace{(x^2+5x)^{-1/2}}_{\text{2nd}}\underbrace{\left(\tfrac{1}{2x}\right)(2)}_{\text{derivative of 1st}}$$

OR using Quotient Rule

$$y' = \frac{(x^2+5x)^{1/2}\left(\frac{1}{2x}\right)(2) - \ln(2x)\left(\frac{1}{2}\right)(x^2+5x)^{-1/2}(2x+5)}{\left((x^2+5x)^{1/2}\right)^2}$$

11 on Test 3

a) $y = \sqrt{\sin(x)} = (\sin(x))^{1/2}$

$$y' = \frac{1}{2}(\sin(x))^{-1/2} \cdot \cos(x)$$

alt quest: $y = \sqrt{\sin(x^2)} = (\sin(x^2))^{1/2}$

$$y' = \frac{1}{2}(\sin(x^2))^{-1/2} \cdot \cos(x^2) \cdot 2x$$

19) a) $f(x) = \overset{1st}{\boxed{e^{2x}}} \cdot \overset{2nd}{\boxed{\sin(3x)}}$

$f'(x) = e^{2x} \cdot (\cos(3x))(3) + (\sin(3x))\, e^{2x} \cdot 2$

$= 3e^{2x} \cdot \cos(3x) + 2e^{2x}(\sin(3x))$

#7 from Test 3

$y = x^3 \ln x$

$\frac{dy}{dx}$ a

$$\frac{d}{dx}\left[\boxed{x^3}\cdot\boxed{\ln x}\right] = 3x^2 \cdot \ln(x) + x^3 \cdot \left(\frac{1}{x}\right)$$

#5 $\frac{d}{dx}\left[e^{x^2}\right] = e^{x^2} \cdot (2x) = 2x \cdot e^{x^2}$